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14. ABSTRACT The objective of this research is to further the understanding of complex dynamical behavior in hybrid systems. Hybrid dynamical systems are of interest to the Air Force because of the high degree of autonomy and cooperation that future reconnaissance and fighter aircraft will maintain. Complex control and coordination algorithms for such vehicles, involving multiple mode switching and other high-level supervisory control architectures, give rise to complicated hybrid dynamical systems with behaviors that can be difficult to predict and understand. The proposed research aims to provide systematic tools for predicting and shaping the behavior of complex hybrid dynamical systems.					
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**FINAL REPORT**  
**COMPLEX DYNAMICAL BEHAVIOR IN HYBRID SYSTEMS**

AFOSR F9550-09-1-0203

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Directorate of Mathematical and Computer Sciences  
February 29, 2012

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## **Summary**

The objective of this research was to further the understanding of complex dynamical behavior in hybrid systems. Hybrid dynamical systems are of interest to the Air Force because of the high degree of autonomy and cooperation that future reconnaissance and fighter aircraft will maintain. Complex control and coordination algorithms for such vehicles, involving multiple mode switching and other high-level supervisory control architectures, give rise to complicated hybrid dynamical systems with behaviors that can be difficult to predict and understand. The proposed research aimed to provide systematic tools for predicting and shaping the behavior of complex hybrid dynamical systems.

We considered several classical dynamical systems concepts that needed further development in the hybrid systems setting. These concepts included principles of time-scale separation, reduced-order behavior on invariant manifolds, and the characterization of complex attractors through the notion of Omega-limit sets. Omega-limit sets play an important role in understanding tracking control, observer design, and output regulation for hybrid systems. In addition, we investigated the utility of classical theorems for the analysis of large-scale systems in the setting of hybrid systems. These ideas include the development and demonstration of interconnection theorems for hybrid systems, including nonlinear small-gain theorems and other dissipativity-based results. We also studied synchronization mechanisms in hybrid systems, which may prove useful for wireless-communication-based military control applications. We also worked to sharpen analysis and synthesis tools for networked control systems, which fit naturally within the hybrid dynamical systems framework. Finally, we considered approximation-based stability results, including a “linearization” principle.

## **Research Publications**

The research supported by this grant resulted in 2 books, 29 published journal papers, 38 refereed conference papers, and 1 book chapter. These publications are listed below.

### Books

B1. L. Zaccarian, A.R. Teel, *Modern anti-windup synthesis: control augmentation for actuator saturation*, Princeton University Press, 2011.

B2. R. Goebel, R.G. Sanfelice, A.R. Teel, *Hybrid Dynamical Systems*, Princeton University Press, 2012.

### Journal papers

J1. L. Marconi and A.R. Teel, ``Matched disturbance suppression for nonlinear systems stabilizable by logic-based feedback'', *Automatica*, 2012, to appear.

J2. C.G. Mayhew, R.G. Sanfelice, J. Sheng, M. Arcak, A.R. Teel, ``Quaternion-based hybrid feedback for robust global attitude synchronization'', *IEEE Transactions on Automatic Control*, 2012, to appear.

J3. W. Wang, D. Nesic, A.R. Teel, ``Input-to-state stability for a class of hybrid dynamical systems via averaging'', *Mathematics of Control, Signals, and Systems*, vol. 23, no. 4, pp. 223-256, 2012.

J4. C.G. Mayhew and A.R. Teel, ``On the topological structure of attraction basins for differential inclusions'', *Systems & Control Letters*, vol. 60, no. 12, pp. 1045-1050, 2011.

J5. R.G. Sanfelice, A.R. Teel, ``On singular perturbations due to fast actuators in hybrid control systems'', *Automatica*, vol. 47, no. 4, pp. 692-701, 2011.

J6. C.G. Mayhew, R.G. Sanfelice, A.R. Teel, ``Quaternion-Based Hybrid Control for Robust Global Attitude Tracking'', *IEEE Transactions on Automatic Control*, vol. 56, no. 11, pp. 2555-2566, 2011.

J7. D. Nesic, A.R. Teel, L. Zaccarian, ``Stability and performance of SISO control systems with First Order Reset Elements'', *IEEE Transactions on Automatic Control*, vol. 56, no. 11, pp. 2567-2582, 2011.

J8. C. Prieur, A.R. Teel, ``Uniting local and global output feedback controllers'', *IEEE Transactions on Automatic Control*, vol. 56, no. 7, pp. 1636-1649, 2011.

J9. A.R. Teel and L. Marconi, ``Stabilization for a class of minimum phase hybrid systems under an average dwell-time constraint'', *International Journal of Robust and Nonlinear Control*, vol. 21, no. 10, pp. 1178-1192, 2011.

- J10. L. Zaccarian, D. Nesic, A.R. Teel, ``Analytical and numerical Lyapunov functions for SISO linear control systems with first-order reset elements'', *International Journal of Robust and Nonlinear Control*, vol. 21, no. 10, pp. 1134-1158, 2011.
- J11. D. Dacic, D. Nesic, A.R. Teel, W. Wang, ``Path Following for Nonlinear Systems with Unstable Zero Dynamics: an Averaging Solution,'', *IEEE Transactions on Automatic Control*, vol. 56, no. 4, pp. 880-886, 2011.
- J12. Y. Or, A.R. Teel, ``Zeno Stability of the Set-Valued Bouncing Ball," *IEEE Transactions on Automatic Control*, vol.56, no.2, pp.447-452, Feb. 2011.
- J13. C. Cai, A.R. Teel, "Output-to-state stability for hybrid systems", *Systems & Control Letters*, Volume 60, Issue 1, January 2011, Pages 62-68.
- J14. R. Goebel and A.R. Teel, ``Pre-asymptotic stability and homogeneous approximations of hybrid dynamical systems'', *SIAM Review*, vol. 52, no. 1, pp. 87-109, 2010.
- J15. T. Hu, T. Thibodeau, A.R. Teel, ``A Unified Lyapunov Approach to Analysis of Oscillations and Stability for Systems With Piecewise Linear Elements'', *IEEE Transactions on Automatic Control*, vol.55, no.12, pp.2864-2869, Dec. 2010.
- J16. A.R. Teel, D. Nesic, ``Averaging for a class of hybrid systems'', *Dynamics of Continuous, Discrete and Impulsive Systems Series A: Mathematical Analysis* 17 (2010) 829-851.
- J17. W.P.M.H. Heemels, A.R. Teel, N. van de Wouw, D. Nesić, ``Networked Control Systems With Communication Constraints: Tradeoffs Between Transmission Intervals, Delays and Performance'', *IEEE Transactions on Automatic Control*, vol.55, no.8, pp.1781-1796, Aug. 2010.
- J18. P. Naghshtabrizi, J.P. Hespanha, A.R. Teel, ``Stability of delay impulsive systems with application to networked control systems'', *Transactions of the Institute of Measurement and Control*, Vol. 32, No. 5, pp. 511-528, 2010.
- J19. A.R. Teel, D. Nesic, A. Loria, E. Panteley, ``Summability characterizations of uniform exponential and asymptotic stability of sets for difference inclusions'', *Journal of Difference Equations and Applications*, vol. 16, no. 2-3, pp. 173-194, 2010.
- J20. R.G. Sanfelice, A.R. Teel, ``Dynamical properties of hybrid systems simulators'', *Automatica*, Volume 46, Issue 2, February 2010, Pages 239-248.
- J21. B. Jayawardhana, E.P. Ryan, A.R. Teel, ``Bounded-Energy-Input Convergent-State Property of Dissipative Nonlinear Systems: An iISS Approach'', *IEEE Transactions on Automatic Control*, vol.55, no.1, pp.159-164, Jan. 2010.

- J22. D. Dai, T. Hu, A. R. Teel, L. Zaccarian, ``Output feedback design for saturated linear plants using deadzone loops'', *Automatica*, vol. 45, no. 12, pp. 2917-2924, 2009.
- J23. M. Lazar, W.P.M.H. Heemels, A.R. Teel, ``Lyapunov Functions, Stability and Input-to-State Stability Subtleties for Discrete-Time Discontinuous Systems'', *IEEE Transactions on Automatic Control*, vol.54, no.10, pp.2421-2425, Oct. 2009.
- J24. R.G. Sanfelice, A.R. Teel, ``Asymptotic Stability in Hybrid Systems via Nested Matrosov Functions'', *IEEE Transactions on Automatic Control*, vol.54, no.7, pp.1569-1574, July 2009.
- J25. D. Dai, T. Hu, A.R. Teel, L. Zaccarian, ``Piecewise-quadratic Lyapunov functions for systems with deadzones or saturations'', *Systems & Control Letters*, vol. 58, no. 5, pp. 365-371, 2009.
- J26. R. Goebel, R.G. Sanfelice, A.R. Teel, ``Hybrid dynamical systems'', *IEEE Control Systems Magazine*, vol.29, no.2, pp.28-93, April 2009.
- J27. D. Nesic, A.R. Teel, D. Carnevale, ``Explicit Computation of the Sampling Period in Emulation of Controllers for Nonlinear Sampled-Data Systems'', *IEEE Transaction on Automatic Control*, vol.54, no.3, pp.619-624, March 2009.
- J28. Rafal Goebel, Christophe Prieur, Andrew R. Teel, ``Smooth patchy control Lyapunov functions'', *Automatica*, vol. 45, no. 3, pp. 675-683, March 2009.
- J29. D. Nešić, A. Loría, E. Panteley, A.R. Teel, ``On stability of sets for sampled-data nonlinear inclusions via their approximate discrete-time models and summability criteria'', *SIAM J. Control Optim.* 48 (2009), no. 3, pp. 1888—1913.

### Conference papers

- C1. M. Heemels, T. Donkers, A.R. Teel, ``Periodic event-triggered control based on state feedback'', *50th IEEE Conference on Decision and Control*, Dec. 2011.
- C2. F. Forni, A.R. Teel, L. Zaccarian, ``Tracking control in billiards using mirrors without smoke, Part I: Lyapunov-based local tracking in polyhedral regions'', *50th IEEE Conference on Decision and Control*, Dec. 2011.
- C3. F. Forni, A.R. Teel, L. Zaccarian, ``Tracking control in billiards using mirrors without smoke, Part II: additional Lyapunov-based local and global results'', *50th IEEE Conference on Decision and Control*, Dec. 2011.
- C4. N. Cox, L. Marconi, A.R. Teel, ``Hybrid output regulation with unmeasured clock'', *50th IEEE Conference on Decision and Control*, Dec. 2011.
- C5. C.G. Mayhew, R.G. Sanfelice, A.R. Teel, ``Further results on synergistic Lyapunov functions and hybrid feedback design through backstepping'', *50th IEEE Conference on Decision and Control*, Dec. 2011.

- C6. A.R. Teel, D. Nesic, "Lyapunov functions for L2 and input-to-state stability in a class of quantized control systems", *50th IEEE Conference on Decision and Control*, Dec. 2011.
- C7. W. Wang, A.R. Teel, D. Nesic, "Novel results in averaging analysis of singularly perturbed hybrid systems", *50th IEEE Conference on Decision and Control*, Dec. 2011.
- C8. R. Skjetne, U. Jorgensen, A.R. Teel, "Line-of-sight path-following along regularly parametrized curves solved as a generic maneuvering problem", *50th IEEE Conference on Decision and Control*, Dec. 2011.
- C9. R. Gielen, M. Lazar, A.R. Teel, "Small-gain results for discrete-time networks of systems with delays", *50th IEEE Conference on Decision and Control*, Dec. 2011.
- C10. W. Wang, D. Nesic, A.R. Teel, "Averaging of hybrid dynamical systems with disturbances", *Australian Control Conference*, pp. 392-397, 2011.
- C11. W. Wang, A.R. Teel, D. Nesic, "Averaging tools for singularly perturbed hybrid systems", *Australian Control Conference*, pp. 88-93, 2011.
- C12. C.G. Mayhew, AR. Teel, "Hybrid Control of Rigid-Body Attitude with Synergistic Potential Functions", *American Control Conference*, pp. 287-292, 2011.
- C13. C.G. Mayhew, AR. Teel, "Synergistic Potential Functions for Hybrid Control of Rigid-Body Attitude", *American Control Conference*, 875-880, 2011.
- C14. C.G. Mayhew, R.G. Sanfelice, A.R. Teel, "Synergistic Lyapunov functions and backstepping hybrid feedbacks", *American Control Conference*, pp. 3203-3208, 2011.
- C15. C.G. Mayhew, R.G. Sanfelice, AR. Teel, "On quaternion-based attitude control and the unwinding phenomenon", *American Control Conference*, pp. 299-304, 2011.
- C16. C.G. Mayhew, R.G. Sanfelice, A.R. Teel, "On the non-robustness of inconsistent quaternion-based attitude control systems using memoryless path-lifting schemes", *American Control Conference*, pp. 1003-1008, 2011.
- C17. N. Cox, A.R. Teel, L. Marconi, "Hybrid output regulation for minimum phase linear systems", *American Control Conference*, pp. 863-868, 2011.
- C18. A.R. Teel, "Asymptotic stability for hybrid systems via decomposition, dissipativity, and detectability," *49th IEEE Conference on Decision and Control*, pp. 7419-7424, 2010.
- C19. L. Marconi, A.R. Teel, "Design of nonlinear regulators from logic-based stabilizers," *49th IEEE Conference on Decision and Control*, pp. 1522-1527, Dec. 2010.

- C20. C.G. Mayhew, A.R. Teel, ``Hybrid control of spherical orientation," *49th IEEE Conference on Decision and Control*, pp. 4198-4203, Dec. 2010.
- C21. F. Forni, A.R. Teel, ``Instability and overshoots of solutions for a class of homogeneous hybrid systems by Lyapunov-like analysis," *49th IEEE Conference on Decision and Control*, pp. 2390-2395, Dec. 2010.
- C22. F. Forni, A.R. Teel, ``Stability for a class of homogeneous hybrid systems by annular Lyapunov analysis," *49th IEEE Conference on Decision and Control*, pp.3289-3294, Dec. 2010.
- C23. C.G. Mayhew, A.R. Teel, ``Global asymptotic stabilization of the inverted equilibrium manifold of the 3-D pendulum by hybrid feedback," *49th IEEE Conference on Decision and Control*, pp.679-684, Dec. 2010.
- C24. L. Marconi, A.R. Teel, ``A note about hybrid linear regulation," *49th IEEE Conference on Decision and Control*, pp.1540-1545, Dec. 2010.
- C25. A.R. Teel, D. Nesić, ``PWM hybrid control systems: averaging tools for analysis and design," *IEEE International Conference on Control Applications (CCA)*, pp.1128-1133, Sept. 2010.
- C26. Y. Or, A.R. Teel, ``Using the set-valued bouncing ball for bounding Zeno solutions of Lagrangian hybrid systems", *Proceedings of IFAC Symposium on Nonlinear Control Systems Design Symposium*, September, 2010.
- C27. A.R. Teel, L. Marconi, ``A note on stabilization for a class of minimum phase hybrid systems", *Proceedings of IFAC Symposium on Nonlinear Control Systems Design Symposium*, September, 2010.
- C28. A.R. Teel, "Observer-based hybrid feedback: a local separation principle," *American Control Conference*, pp. 898-903, 2010.
- C29. C.G. Mayhew, R.G. Sanfelice, M. Arcak, A.R. Teel, ``Robust global asymptotic attitude synchronization by hybrid control," *American Control Conference*, pp.3666-3671, 2010.
- C30. C.G. Mayhew, A.R. Teel, ``Hybrid control of planar rotations," *American Control Conference*, pp.154-159, 2010.
- C31. D. Dai, T. Hu, A.R. Teel, L. Zaccarian, ``Output feedback synthesis for sampled-data system with input saturation", *American Control Conference*, pp.1797-1802, 2010.
- C32. M. Hartman, N. Bauer, A.R. Teel, ``A hybrid algorithm for finite time parameter estimation", *American Control Conference*, pp. 2176-2181, 2010.

- C33. C.G. Mayhew, R.G. Sanfelice, A.R. Teel, ``Robust global asymptotic attitude stabilization of a rigid body by quaternion-based hybrid feedback'', *Proceedings of the 48th IEEE Conference on Decision and Control*, pp.2522-2527, 15-18 Dec. 2009.
- C34. C.G. Mayhew, R.G. Sanfelice, A.R. Teel, ``Robust global asymptotic stabilization of a 6-DOF rigid body by quaternion-based hybrid feedback'', *Proceedings of the 48th IEEE Conference on Decision and Control*, pp.1094-1099, 2009.
- C35. A.R. Teel, ``Preliminary results on the existence of continuous Lyapunov functions for semicontinuous, stochastic discrete-time systems'', *Proceedings of the 48th IEEE Conference on Decision and Control*, pp.4729-4734, 2009.
- C36. W.P.M.H. Heemels, D. Nesic, A.R. Teel, N. van de Wouw, ``Networked and quantized control systems with communication delays'', *Proceedings of the 48th IEEE Conference on Decision and Control*, pp.7929-7935, 2009.
- C37. A.R. Teel, D. Nesic, A. Loria, E. Panteley, ``Uniform stability of sets for difference inclusions under summability criteria'', *Proceedings of the 48th IEEE Conference on Decision and Control*, pp. 4131-4136, 2009.
- C38. T. Hu, T. Thibodeau, A.R. Teel, ``Analysis of oscillation and stability for systems with piecewise linear components via saturation functions'', *American Control Conference*, pp.1911-1916, June 2009.

## **Book Chapters**

- BC1. W. Wang, A.R. Teel, D. Nesic, ``Averaging results pertaining to the implementation of hybrid feedback via PWM control'', In *Dynamics and Control of Switched Electronics Systems*, F. Vasca and L. Iannelli eds. Springer, 2012.

## Research Accomplishments

*Time-scale separation:* We started with averaging [J16] and singular perturbation [J5] results and followed that with new results that combine singular perturbation and averaging results for hybrid systems [J3, C7, C10, C11, C25, BC1]. All of the new results are corollaries of robustness of asymptotic stability for hybrid systems, a result that was established under a previous research effort supported by AFOSR. The new results demonstrate the efficacy of hybrid feedback implemented through fast actuators whose steady-state behavior is highly oscillatory, like flapping wings of an insect or bird.

*Local asymptotic stability theory based on homogeneous approximations:* Early in this grant, we developed the theory of local asymptotic stability in hybrid dynamical systems based on asymptotic stability of a homogeneous approximation at the operating point [J14], generalizing to hybrid systems the classical result of Lyapunov on asymptotic stability based on the linear approximation for nonlinear differential and difference equations. In follow-up work [C21, C22], we developed numerical, Lyapunov-based algorithms for establishing asymptotic stability of homogeneous approximations.

*Hybrid output regulation:* We began our investigation of output regulation in the presence of a hybrid exosystems in [C24], focusing on the case of relative degree one linear plants. We followed that with solutions for higher relative degree linear systems [C17] and for the situation where the clock variable that determines when the jumps of the disturbances occur is unmeasured [C4]. Hybrid exosystems can be used to model square-wave disturbances and other discontinuous disturbances. In addition to output regulation for discontinuous disturbances, we showed how to incorporate output regulation ideas into controllers that use logic-based feedback [J1, C19]. Some of the results in this area rely on the technical results established in [J9, C27].

*Hybrid feedback control:* A strong emphasis of our research was to develop hybrid feedbacks to achieve global asymptotic stabilization for systems evolving on compact manifolds, a setting where classical feedback cannot achieve robust solutions [J4]. Our work in this direction, which is grounded in our concept of synergistic potential functions and synergistic Lyapunov functions, appears in [C30] for planar rotations, [C20] for spherical orientation, [C23] for the 3D pendulum, [J6, C12, C13, C15, C16, C33] for rigid-body attitude, including rigid-body attitude synchronization [J2, C29], and rigid-body dynamics [C34]. Noteworthy as well is the application of the classical backstepping control idea to hybrid feedbacks, as we developed in [C14] and [C5]. In addition, we have developed new results on reset control systems [J7, J10], and have developed an algorithm for uniting local and global controllers [J8]. We also developed results on tracking in mechanical systems with impacts [C2, C3]. In particular, we have demonstrated how to achieve global asymptotic tracking with arbitrarily small feedback controls in rectangular billiards. Moreover, we established a local separation principle for the design of hybrid state feedbacks and hybrid observers [C28]. We also refined and analyzed a hybrid, finite-time parameter estimator in [C32]. In addition, we initiated new results on periodic event-triggered control algorithms [C1].

*Analysis:* In [C18], we developed dissipativity and decompositions (an alternative viewpoint to interconnections) in our hybrid systems framework. Our focus was on explicit applications of dissipativity concepts, decompositions of hybrid systems to make dissipativity applicable, and also the use of detectability concepts to draw asymptotic stability conclusions. The latter aspect is especially novel since it relies on recent results for hybrid systems that are tied to our contributions on robustness. In [J12] and [C26], we performed a complete Zeno and Lyapunov analysis of a set-valued bouncing ball model which serves as a surrogate for a large class of Euler-Lagrange systems with impacts. We also contributed novel results on the analysis of networked and quantized control systems with communication delays [J27, J18, J17, C36]. We demonstrated various tradeoff curves related to maximum allowable transfer interval, quantization levels, and communication delays. We gave a comprehensive Lyapunov analysis of a zoom-in zoom-out quantization protocol, which results in a hybrid system, in [C6]. Furthermore, we extended the notion of output-to-state stability to hybrid systems and have given equivalent characterizations in [J13]. We provided a novel analysis of simulators for hybrid systems [J20], demonstrating through robustness results for hybrid systems that “consistent” simulators will exhibit semiglobal practical asymptotic stability for compact sets that are globally asymptotically stable for the hybrid system being simulated. We also extended to hybrid systems an analysis tool [J24] we promoted previously for nonlinear systems, based on the notion of Matrosov functions, as an alternative to the invariance principle.

*Additional work:* In addition to our work on hybrid systems, we continued to make contributions in the areas of control synthesis for systems with input saturation [J22, J25, C31], the analysis of set-valued discrete-time systems [J19, J23, J29, C9, C37], path-following algorithms [J11, C8], and the analysis of nonlinear systems with sustained oscillations [J15, C38].

*Pedagogical contributions:* During the grant period, we published two books, one on anti-windup synthesis [B1] and one on hybrid dynamical systems [B2]. These manuscripts are a direct result of research supported by AFOSR, during the current period and under previous grants. In addition, we published one tutorial article [J26] that received the 2010 *IEEE Control Systems Magazine* Best Paper Award.

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## **Personnel Supported**

*Faculty:* Dr. Andrew R. Teel (PI);

*Graduate Student Researchers:* Christopher Mayhew, Nicholas Cox, Matthew Hartman, Andrew Berman, Siladitya Dey

## **Transitions**

none

## **Lifetime honors/awards**

IFAC Fellow, 2010

IEEE Fellow, 2002

AACC Donald P. Eckman Award, 1999

SIAM Control and Systems Theory Prize, 1998

## **Honors/awards during grant**

2010 IFAC Fellow

2010 *IEEE Control Systems Magazine* Best Paper Award

2011 *American Control Conference* Student Best Paper Award